

PHYS-3202 Homework 6 Due 30 Oct 2019

This homework is due in the dropbox outside 2L26 by 10:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. Collisional Cross Section of the Earth

Consider meteoroids approaching the earth (with positive asymptotic speed v).

- Find the total cross section σ for the asteroids to collide with the earth (that is, for an asteroid to approach closer to the earth than the earth's radius R) as a function of the asteroid's asymptotic speed. Ignore friction due to the earth's atmosphere and acceleration around the sun.
- How much larger is this cross section than the earth's geometric cross section πR^2 ? Give your answer as a fractional difference (ie, $(\sigma - \pi R^2)/\pi R^2$) first in terms of the earth mass and radius and the meteoroid's asymptotic speed and then as a dimensionless number for a typical meteoroid speed of $v = 20$ km/s (relative to earth). Does gravity make a significant difference in the likelihood of a meteoroid hitting the earth? You may find astronomical data at the Particle Data Group. Use the nominal equatorial radius for the earth.

2. Pendulum in a Bus

A pendulum of length l hangs from the ceiling of a bus, which is waiting at a red stoplight (stationary with respect to the earth). At time $t = 0$, the light turns green, and the bus starts driving with constant horizontal acceleration \vec{a} . Describe the pendulum's position by θ , the angle from the downward vertical (parallel to the Earth's gravitational acceleration \vec{g}).

- Find the equilibrium position θ_0 of the pendulum when the bus is accelerating. Describe how you find your answer from the perspective of both an inertial observer on the ground and an accelerating observer on the bus.
- Describe the motion of the pendulum if it is hanging straight down at the time the bus starts accelerating. Assume $|\vec{a}| \ll g$.

3. A Rotating Frame by Matrix Multiplication *simplified from KB 5.19*

Consider 3 fixed unit vectors $\hat{i}, \hat{j}, \hat{k}$; a position vector has components x', y', z' along the corresponding axes. Meanwhile, the unit vectors $\hat{x}, \hat{y}, \hat{z}$ are rotated an angle ϕ around the \hat{k} axis. The components of the position vector measured along these axes are x, y, z . These are two descriptions of the same vector \vec{r} . We know that the components are related by the matrix multiplication

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}. \quad (1)$$

We will consider this relationship when ϕ depends on time. Below, define $\vec{\omega} = \dot{\phi} \hat{k}$. Note that these frames have the same origin.

- First, invert the relationship, so we can find the fixed-axis components in terms of the rotating-axis components.
- By taking time derivatives, show that $d\vec{r}/dt = \dot{r} + \vec{\omega} \times r$. In matrix notation, $\dot{\vec{r}} = [\dot{x}, \dot{y}, \dot{z}]^T$, while $d\vec{r}/dt$ is $[\dot{x}', \dot{y}', \dot{z}']^T$ multiplied by the rotation matrix as in (1). The reason for the matrix multiplication is that we must compare components with respect to the same axes.

Likewise, make sure that the cross product term has \vec{r} written in terms of the same rotating components.

4. **Motion Through Parallel EM Fields** *clarified from KB 5.11*

Consider particles of charge q and mass m entering a region of parallel uniform electric and magnetic fields $\vec{E} = E\hat{k}$, $\vec{B} = B\hat{k}$ at $x = y = z = 0$ with initial velocity $\vec{v} = v\hat{i}$. They then strike a screen in the yz plane at $x = a$ (which fluoresces, so we can measure where they hit). The distance $a \ll mv/qB$ is small.

- (a) Write Newton's second law for all three coordinates of one of the charged particles.
- (b) First, find the solutions $x(t), y(t), z(t)$ for the motion of the particles. Then argue that the time traveled before hitting the screen is small and make appropriate approximations to simplify your expressions. *Hint:* you can use solutions from the class notes where appropriate.
- (c) Find the y, z positions where the charged particles hit the screen as a function of v . Then eliminate v to find the z position as a function of the y position. This gives the curve that where particles will hit the screen if there is a spread in velocities v . *Hint:* your final answer should be a parabola.