AdS/CFT Homework 5 Due 1 Dec 2015

This homework is due in the class meeting on the due date You may consult with each other but must write up your solutions on your own; use of on-line solutions is *not allowed.* Contact me if you would like office hours to discuss these problems.

1. **Gauge Group for D-branes at Orbifolds**

   This will try to guide you through how we know what the gauge group is on a stack of D-branes, including the case with an orbifold.

   (a) Consider a case with N D-branes labeled by $i = 1, \cdots N$. If we ignore the state of the string oscillators, the state of an open string is specified by which D-brane each end of the string attaches to. For example, the state $|i, j\rangle$ is for a string that starts at D-brane i and ends on D-brane j (where "start" and "end" refer to the value of the worldsheet spatial coordinate). It is more convenient, however, to use a basis $|\lambda\rangle = \lambda_{ij}|i, j\rangle$ for $N \times N$ matrices λ. These matrices are called *Chan-Paton factors* (or CP factors for short).

      i. Assuming the inner product $\langle i', j'|i, j\rangle = \delta_{i,i'}\delta_{j,j'}$, what is the inner product $\langle\lambda'|\lambda\rangle$? (Remember that the adjoint includes a complex conjugate.)

      ii. Suppose we make a change of basis $\lambda \to U\lambda U^{-1}$. Show that your inner product is compatible with $U \in U(N)$.

   Assuming that the D-branes are all stacked at the same position (and are the same dimensionality, etc), states with a single string oscillator excitation parallel to the branes are massless gauge bosons that therefore obey a $U(N)$ gauge symmetry. The gauge boson states must have CP matrices that are even under the orbifold reflection since the oscillator state is even.

   (b) Now consider a case with N D-branes near a $\mathbb{Z}_2$ orbifold fixed plane (of the same dimensionality of the branes) plus their N image branes. In this case, the indices on the CP factor matrices run over $i = 1, \cdots 2N$, and strings with one oscillator excitation parallel to the branes are still gauge bosons.

      i. Argue that the action of the orbifold reflection on the CP factors can be represented by $\lambda \to \gamma_R\lambda\gamma_R^{-1}$ with $\gamma_R^2 = 1$.

      ii. Suppose we split the CP indices i into the "brane indices" $a = 1, \cdots N$ and "image brane" indices $\hat{a} = 1, \cdots N$. Then $|a, b\rangle$ and $|\hat{a}, \hat{b}\rangle$ states are strings stretching just between branes or image branes, while $|a, \hat{b}\rangle$ and $|\hat{a}, b\rangle$ strings stretch from branes to image branes and vice versa. If the branes are away from the orbifold fixed point, the latter types of states are massive and cannot contribute to the gauge group. Argue that the reflection matrix and even CP matrices for the massless strings (in this basis) are given by the block-diagonal form

$$\gamma_R = \begin{bmatrix} 0_N & 1_N \\ 1_N & 0_N \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda' & 0_N \\ 0_N & \lambda' \end{bmatrix}.$$

      Here, 1_N is the N-dimensional identity, etc. Then argue that the change of basis $U\lambda U^{-1}$ that preserves the CP matrix form is the block diagonal $U = \mathrm{diag}(U', U')$, where $U' \in U(N) \subset U(2N)$. This shows that, despite the "extra" image branes, the gauge group is the same as without the orbifold in this case.

(c) Now suppose that the branes are sitting on top of the orbifold fixed plane, so all the gauge boson states are massless. In this case, it is convenient to work in a basis for the CP matrices such that $\gamma_R = \text{diag}(1_N, -1_N)$ is block diagonal. Find the form of the allowed CP matrices in this basis and argue that the change of basis matrix that preserves this form is the block diagonal $U = \text{diag}(U_1, U_2) \in U(2N)$ for U_1, U_2 distinct $U(N)$ transformations. Therefore, the gauge group is enhanced to $U(N) \times U(N)$ when the branes are at the orbifold.

(d) Finally, suppose that we have branes at the orbifold fixed point and take $\gamma_R = \text{diag}(1_{N+M}, -1_N)$. Repeat the argument of part (c) to show that the gauge group is $U(N+M) \times U(N)$. These extra M branes are called fractional branes because they have no image and cannot be moved away from the orbifold plane.