AdS/CFT Homework 4 Due 10 Nov 2015

This homework is due in the class meeting on the due date You may consult with each other but must write up your solutions on your own; use of on-line solutions is *not allowed*. Contact me if you would like office hours to discuss these problems.

1. **Surface Gravity Revisited**

   Consider the near horizon limit for a black hole metric

   $$ds^2 = -\alpha^2\rho^2 dt^2 + d\rho^2 + \cdots \ . \tag{1}$$

   Supposedly, α will be equal to the surface gravity κ, but recall that we had some trouble in class using the definition $\xi^b\nabla_b\xi^a = \kappa\xi^a$ with the metric (1) and Killing vector $\xi^a = (1, 0, \cdots)$ (which is appropriate for a Schwarzschild black hole). From consultation with Gabor, the problem seems to be that both sides of this equation are vectors — coordinate dependent — and the $\rho \to 0$ limit is singular in these coordinates.

   However, it is possible to show that this definition of κ is equivalent to $\kappa^2 = -\nabla_a\xi_b\nabla^a\xi^b/2$ (see Wald's GR text, for example, for a proof). Use this definition to show that $\kappa = \alpha$.

2. **Temperature of Poincaré AdS Horizon** *shortened version of a problem by McGreevy*

   Consider the Euclidean AdS-Schwarzschild metric in the Poincaré patch (in 5D)

   $$ds^2 = \frac{1}{z^2}\left(-f(z)d\tau^2 + \frac{dz^2}{f(z)} + d\vec{x}^2\right) \ , \quad f(z) = 1 - \frac{z^4}{z_0^4} \ . \tag{2}$$

   Go to the near-horizon limit $z \sim z_0$ and find the periodicity of τ necessary to avoid a conical singularity. What is the temperature of this black hole?

3. **Small and Large Black Holes in Global AdS**

   The AdS-Schwarzschild metric

   $$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \ , \quad f(r) = \left(1 - \frac{r_0^2}{r^2} + \frac{r^2}{R^2}\right) \tag{3}$$

   is asymptotically AdS with curvature radius R in global coordinates in 5D. The horizon occurs at radius r_+, which is the largest root of $f(r)$. Note that $d\Omega^2$ is the metric on a sphere and that the black hole mass (above the vacuum energy of empty AdS) is $M \propto r_0^2$.

   (a) By using the method of one of the previous problems, find the black hole temperature. You may write your answer in terms of r_0, r_+, R, as you find convenient for the rest of the problem.

   (b) For small black holes satisfying $r_+ \ll R$, find the temperature and entropy as a function of black hole mass (ie, r_0^2).

   (c) Repeat the previous part for large black holes $r_+ \gg R$. Using your results, sketch the black hole temperature as a function of mass. What do these results mean for the ability of an AdS black hole to exist in equilibrium with radiation?