

PHYS-4601 Homework 17 Due 5 Mar 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. Not-Quite-Square Well

Consider a particle moving in a 1D well of potential

$$\begin{cases} V_0 x/a & 0 < x < a \\ \infty & \text{otherwise} \end{cases} . \quad (1)$$

Assume that $\epsilon = ma^2V_0/\hbar^2 \ll 1$. Recall that the energy eigenfunctions and eigenvalues are

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a}\right)^2, \quad n = 1, 2, \dots . \quad (2)$$

- (a) Show that the first order contribution to the energy is $E_n^1 = V_0/2$ for all n .
- (b) Now consider the ground state of the system. Recalling that the first order correction to the ground state can be written as

$$|\psi_1^1\rangle = \sum_{n=2}^{\infty} c_n |\psi_n^0\rangle, \quad (3)$$

use Maple's `seq` and `int` commands to make a list of the coefficients c_n for $n = 2, \dots, 10$. Attach a copy of your Maple code. You should work in units where $a = 1$ and express your answer in terms of the parameter ϵ .

- (c) Use Maple to plot the uncorrected ground state wavefunction and the wavefunction with first order terms (including corrections from the $n = 2, \dots, 10$ states) on the same plot. In order to see the difference, use an exaggerated value of $\epsilon = 3$.

2. Stark Effect based on Griffiths 6.36

The presence of an external electric field $E_0 \hat{z}$ shifts the energy levels of a hydrogen atom, which is called the Stark effect. Consider the hydrogen atom to be described by the Coulomb potential; the external electric field introduces a perturbation

$$H_1 = eE_0 z = eE_0 r \cos \theta . \quad (4)$$

We have already seen on homework that the expectation value of this Hamiltonian in the ground state $n = 1$ vanishes, so there is no shift in the ground state energy. In this problem, we consider the degenerate perturbation theory of the $n = 2$ states. *As spin does not enter, do not consider it in this problem.*

- (a) The four states $|2, 0, 0\rangle$, $|2, 1, 0\rangle$, and $|2, 1, \pm 1\rangle$ are degenerate at 0th order. Label these states sequentially as $i = 1, 2, 3, 4$. Show that the matrix elements $W_{ij} = \langle i | H_1 | j \rangle$ form the matrix

$$W = -3aeE_0 \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (5)$$

where empty elements are zero and a is the Bohr radius. *Hint:* Note that L_z commutes with H_1 , so only states with the same quantum number m can have nonzero matrix elements; this will save you quite a bit of work. Then use the angular wavefunctions to see that all the diagonal elements of W must vanish. Finally, use the explicit wavefunctions to evaluate the remaining matrix elements of W (there should only be one independent one left).

- (b) Diagonalize this matrix to show that $|\pm\rangle = (1/\sqrt{2})(|2, 0, 0\rangle \pm |2, 1, 0\rangle)$ are eigenstates of W . Find the first order shift in energies of $|\pm\rangle$. *Hint:* Note that the corrected eigenstates may still have contributions from other values of the principal quantum numbers n , but that doesn't quite matter.
- (c) Finally, show that the states $|\pm\rangle$ have a nonzero dipole moment $p_z = -e\langle z\rangle$ and calculate it. You should not need to do any more calculations; just use your answer from part (b).

3. Matrix Perturbation Theory

Consider the matrix Hamiltonian

$$H \simeq \begin{bmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{bmatrix}. \quad (6)$$

- (a) To first order in perturbation theory, find the energy eigenvalues and eigenstates.
- (b) Find the energy eigenvalues and eigenstates exactly in the case that $E_1 = E_2$. Then expand them as a power series in ϵ and compare to your perturbative answer from part (a). If you find a difference, explain why and demonstrate the resolution.