

PHYS-4601 Homework 14 Due 5 Feb 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. Finishing Bell's Inequality *Griffiths 4.50*

Consider 2 distinguishable spin 1/2 particles in the singlet ($s = 0$) total spin state. If \vec{a} and \vec{b} are two unit vectors, show that

$$\left\langle \left(\vec{a} \cdot \vec{S}^{(1)} \right) \left(\vec{b} \cdot \vec{S}^{(2)} \right) \right\rangle = -\frac{\hbar^2}{4} \vec{a} \cdot \vec{b}. \quad (1)$$

Hint: Think about a convenient choice of axes.

2. Physical Representation of Quantum Gates

- Consider a qbit represented by a charged spin-1/2 particle (such as an electron or proton) so that the bit $|0\rangle$ is spin up and $|1\rangle$ is spin down. Write the Hadamard gate operator as a matrix in the usual basis.
- Then show that this operator is the time evolution operator for the charged particle first exposed to the z component of a magnetic field for an appropriate length of time and then exposed to the y component for the right length of time. You may find a physically irrelevant overall phase.
- Consider the same physical representation of a qbit and show that the quantum NOT gate can be implemented by the time evolution operator of the particle in the x component of a magnetic field for an appropriate length of time (up to an unphysical overall phase).

3. 2-Qbit Gates

Consider a 2 qbit system. Choose a basis for the 2 qbit Hilbert space and use it for all parts of this problem.

- Write the CNOT gate operator as a matrix in that basis and show that it is unitary.
- Consider the 1 qbit gate NOT acting only on the first qbit of our two. Write this gate (call it NOT₁) as a matrix in your 2-qbit basis.
- inspired by Blümel 7.5.4* We can create a new quantum gate G by first acting with the NOT₁ and then CNOT. Give an example of an input 2-qbit state that can be factorized (that is, written as $|\psi\rangle_1|\phi\rangle_2$ for some 1-qbit states $|\psi\rangle, |\phi\rangle$) that is turned into an entangled state by G ($G(|\psi\rangle_1|\phi\rangle_2)$ cannot be factorized).