PHYS-4601 Homework 12 Due 15 Jan 2015

This homework is due in the dropbox outside 2L26 by 11:59PM on the due date. You may alternately email a PDF (typed or black-and-white scanned) or give a hardcopy to Dr. Frey.

1. **Quantum Earth** *almost Griffiths 4.17*

   In this problem, treat the earth-sun system as an analog of the hydrogen atom. Let M be the mass of the sun and m the mass of the earth.

   (a) By comparing the Newtonian gravitational potential to the Coulomb potential of the hydrogen atom, write down the gravitational Bohr radius a_g and the quantum mechanical earth-sun energy E_n in terms of M, m, and the Newton constant G.

   (b) Compare the classical energy of a planet in a circular orbit of radius r to your formula E_n and show that $n = \sqrt{r/a_g}$. Estimate n for the earth. Let r be 1 astronomical unit. Just give one significant digit. *Hint*: You can look up all the astrophysical data you need at `http://pdg.lbl.gov/2014/reviews/rpp2014-rev-astrophysical-constants.pdf`. It also helps to remember the virial theorem, which says that kinetic energy is $-1/2$ potential energy for an orbit in a $1/r$ potential.

   (c) Show that the total orbital angular momentum quantum number ℓ is approximately n; that is, ℓ is close to its maximum allowed value.

   (d) *related to Griffiths 4.46* We can now relate our estimated results to quantum mechanics. Start by using the recursion relation discussed in class for the radial wavefunction to show that the radial wavefunction for a state with $\ell = n - 1$ is $R \propto r^{n-1} \exp[-r/na_g]$. Find the normalization constant and then $\langle r \rangle$. For large n, show that this agrees with your previous result $n = \sqrt{r/a_g}$.

2. **Hydrogen-Like Atoms**

   A hydrogen-like atom is one that describes the motion of a single particle (typically an electron) around a central Coulomb potential. For example, the deuterium atom from the previous assignment and the quantum earth above are both hydrogen-like atoms. The hydrogen wavefunctions still apply, just with modifications due to possibly different charges or reduced masses.

   (a) *based on Griffiths 4.13* Find $\langle r^2 \rangle$ for the ground state of a hydrogen-like atom in terms of the Bohr radius. Find the ratio of this result between hydrogen to that for a helium ion, which has a single electron orbiting a nucleus of charge $+2e$. (In other words, find $\langle r^2 \rangle_H / \langle r^2 \rangle_{He^+}$.) What does this mean about the comparative "size" of these two atoms?

   (b) *Positronium* is a hydrogen-like atom formed as a bound state of an electron and a positron (anti-electron). A positronium atom can decay into photons if the electron and positron "hit" each other; as a result, the decay rate is proportional to $|\psi(r=0)|^2$, where ψ is the wavefunction as a function of the relative separation r (and the angles θ, ϕ). What is the ratio of the decay rates of positronium atoms in the $n = 1, \ell = 0$ and $n = 2, \ell = 0$ states? Ignore spin.

3. **3-Particle States** *a mix of Griffiths 5.7 and 5.33*

   Consider three particles, each of which is in one of the single-particle states $|\alpha\rangle$, $|\beta\rangle$, or $|\gamma\rangle$, which are orthonormal.

(a) If the particles are bosons, write down the state where one particle is in each of $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$. *Hint*: This state must be symmetric under the exchange of *any* pair of the bosons.

(b) Write down all possible 3-particle states (including normalization) with two particles in the same 1-particle state and the third particle in a different 1-particle state, still in the case that the particles are indistinguishable bosons.

(c) How many linearly independent states can you form if the particles are fermions? Write down all the possible linearly independent states. *Hint*: Similarly to the above, these states must be antisymmetric under the exhange of *any* pair of the fermions.