

Quantum Mechanics II PHYS-4601

First In-Class Test

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Instructions:

- Do not turn over until instructed.
- You will have 75 minutes to complete this test.
- No electronic devices or hardcopy notes are allowed.
- INSTRUCTIONS REGARDING THE QUESTIONS WILL GO HERE.
- **Only the lined pages of your exam book will be graded. Use the blank pages for scratch work only.**

Useful Formulae:

- Schrödinger Equation
 - time-dependent and position-basis time-independent

$$i\hbar \frac{d}{dt} |\Psi\rangle = H |\Psi\rangle, \quad \langle \vec{x} | H | \psi \rangle = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{x}) + V(\vec{x}) \psi(\vec{x}) = E \psi(\vec{x})$$

- Ehrenfest's Theorem and time-evolution operator

$$\frac{d\langle \mathcal{O} \rangle}{dt} = \frac{i}{\hbar} \langle [H, \mathcal{O}] \rangle + \left\langle \frac{\partial \mathcal{O}}{\partial t} \right\rangle, \quad e^{-iHt/\hbar}$$

- Radial equation

$$-\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + \left[V(r) + \frac{\hbar^2 \ell(\ell+1)}{2m r^2} \right] u = E u, \quad u(r) = r R(r)$$

- Momentum and Position
 - Inner product of eigenstates (change of basis)

$$\langle x | p \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \quad \text{or} \quad |p\rangle = \int dx \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} |x\rangle$$

in 1D (generalizing to a dot product for vectors in 3D)

- Operator commutator $[x, p] = i\hbar$
- Operator representation in position basis

$$\langle x | p | \psi \rangle = -i\hbar \frac{d\psi}{dx} \Leftrightarrow p \simeq -i\hbar \frac{d}{dx}$$

- Uncertainty
 - The uncertainty $\sigma_{\mathcal{O}}$ obeys $\sigma_{\mathcal{O}}^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$
 - Heisenberg Uncertainty Principle $\sigma_A \sigma_B \geq (1/2) |\langle [A, B] \rangle|$

- Delta Functions in Potentials $V(x) = -\alpha\delta(x - a) + \dots$

$$\lim_{\epsilon \rightarrow 0} \left(\frac{d\psi}{dx}(a + \epsilon) - \frac{d\psi}{dx}(a - \epsilon) \right) = -\frac{2m\alpha}{\hbar^2} \psi(a)$$

- Infinite Square Well $V(x) = 0$ for $-a < x < a$, $V(x) = \infty$ otherwise
 - Eigenfunctions $\psi_n(x) = (1/\sqrt{a}) \cos(n\pi x/2a)$ (n odd) or $\psi_n(x) = (1/\sqrt{a}) \sin(n\pi x/2a)$ (n even)
 - Energy Eigenvalue $E_n = (\hbar^2/2m)(n\pi/2a)^2$

- Probability current $\vec{j} = \frac{i\hbar}{2m} (\Psi \vec{\nabla} \Psi^* - \Psi^* \vec{\nabla} \Psi)$

- 1D Harmonic Oscillator $V(x) = (1/2)m\omega^2 x^2$

- Stationary State Wavefunctions (see Hermite polynomials below)

$$\langle x|n \rangle = \psi_n(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n \left(\sqrt{\frac{m\omega}{\hbar}} x \right) \exp \left[-\frac{m\omega x^2}{2\hbar} \right]$$

- Ladder operators

$$a = \sqrt{\frac{m\omega}{2\hbar}} x + i \frac{p}{\sqrt{2\hbar m\omega}}, \quad a|n \rangle = \sqrt{n}|n-1 \rangle, \quad a^\dagger|n \rangle = \sqrt{n+1}|n+1 \rangle, \quad [a, a^\dagger] = 1$$

- Hamiltonian $H = \hbar\omega (a^\dagger a + 1/2)$

- Possibly useful integrals

- Gaussian integral

$$\int_{-\infty}^{\infty} dx e^{-ax^2+bx} = \sqrt{\frac{\pi}{a}} e^{b^2/4a}$$

where a, b can be complex as long as $\text{Re } a > 0$

- Exponential integrals

$$\int_0^{\infty} dx x^p e^{-x/b} = p! b^{p+1}$$

TABLE 2.1: The first few Hermite polynomials, $H_n(\xi)$.

$H_0 = 1,$ $H_1 = 2\xi,$ $H_2 = 4\xi^2 - 2,$ $H_3 = 8\xi^3 - 12\xi,$ $H_4 = 16\xi^4 - 48\xi^2 + 12,$ $H_5 = 32\xi^5 - 160\xi^3 + 120\xi.$
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